Statistical Methods

— OBJECTIVES —

This lecture provides an introduction to the main families of statistical methods. In a first chapter, the lecture covers historical developments of statistical inference, from classical frequentist inference to recent machine learning and AI methods. The second chapter deals with statistical estimation, whose goal is to measure some quantity of interest from noisy data, for instance the flux of a star from an image or the duration of an exoplanet transit from a photometric time series. Statistical estimation also aims to provide confidence interval on the results and guarantees on the method used. The third chapter deals with statistical detection, whose goal is to decide whether a specific signal is present or not in the data and to provide a measure of confidence in the result. The last chapter focuses on Bayes estimation, where the concept or probability is extended to incorporate prior belief in the inference process. This lecture combines analytical and numerical exercises (in Python).

— EVALUATION —

1. On each chapter, a "friendly quiz" is provided, along with a correction of the analytical and numerical exercises (in Python). On the Discord server of the course, each student has a personal channel on which the solutions of the exercises and questions can be posted. For each of the four chapters, a noted quiz session evaluates the students' level. The average of the four quizs' grades makes a grade A.
2. A final, 3h written exam on all chapters provides a grade B. This exam is based on the exercises seen in the lectures.

The final grade is $(A + B)/2$.

— MAIN PROGRESSION STEPS —

All sessions will be split between lectures and practical work. Lectures notes for each chapter are provided before the sessions, along with quizs allowing to grasp the main concepts. Sessions are dedicated to answer the questions and to practical works. Practical works include written and numerical exercises (50% each).

— BIBLIOGRAPHY & RESOURCES —

- Lecture notes, slides, python codes and data available online.
- "Computer Age Large Scale Inference", B. Efron, Cambridge University Press, 2019
- "Statistics, Data Mining & Machine Learning in Astronomy", Z. Ivezic, Princeton Series in Modern Observational Astronomy, Second Edition, 2020
- "Fundamentals of Statistical Signal Processing, Volume I: Detection Theory", S. Kay, Prentice Hall, 1993
- "Fundamentals of Statistical Signal Processing, Volume I: Estimation Theory", S. Kay, Prentice Hall, 1993

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Content

- Chapter 1 provides a short history of statistical inference. It makes an important difference between algorithmic developments (that are driven by or focused on applications) vs inferential arguments (that tend to support the methodology). Statistical Inference has developped in three stages that will be studied in this lecture:
  1. Great themes of classical inference : Bayesian, frequentist, and Fisherian (Maximum Likelihood Estimation).
  2. Early computer-age developments, from the 1950s through the 1990s. Many progresses have been brought by technology (bootstrap, jackknife, cross-validation, empirical Bayes, MCMC - to be seen later in this lecture). This was the early rise of machine learning.

3. "XX1 century topics": the era of ambitious algorithms of machine learning, data mining and AI (Artifical Intelligence). These algorithms always involve and combine classical algorithms and concepts. The justification and understanding of such algorithms is the ongoing task of modern inference. Hence, the main concepts and tools of classical inference must be known in order to understand and use AI and machine learning. (The picture of this Syllabus was generated by DALL·E by the way).

Important concepts are introduced through three application examples on regression, bootstrap confidence intervals and hypothesis testing.

- Chapter 2 deals with statistical estimation, which is the process of inferring the values of parameters of interest from data, and providing confidence levels for the result.

  Fundamentals quantities and properties of estimators are studied : bias, variance, Mean Square Error, efficiency, and optimality of estimators. The Cramer-Rao lower Bound, that characterizes the minimum variance of any unbiased estimator, is studied as well as the Fisher information.

  The last part of the chapter deals with one of the most used estimator, the Maximum Likelihood estimator, and its performance (asymptotic efficiency and distribution).

- Chapter 3 deals with statistical detection, which provides methods for deciding whether some specific signal is hidden or not in the data, and providing estimation on the significance level of the result.

  This Chapter first introduces general concepts related to detection theory : statistical test, test statistic, statistical hypotheses, probability of false alarm and of detection, p-values and ROC (Receiver Operating Characteristics) curves.

  The first important testing procedure that is studied is the Likelihood Ratio (or Neyman-Pearson) test. This is the most powerful detection procedure, which requires all parameters of the statistical model to be known. This test serves as a very useful benchmark to any practical testing approach.

  In real applications, some parameters are unknown. In this case, a useful and often powerful testing approach is the Generalized Likelihood Ratio, where the Maximum Likelihood Estimates of the parameters are plugged in the Likelihood Ratio to mimic the Neyman-Pearson test.

- Chapter 4 turns to Bayesian estimation. Bayes' rule is first introduced along with historical aspects.

  The central idea of Bayes estimation is to incorporate prior knowledge in the inference process through a prior distribution on the unknown parameters. If we want to measure the mass of a planet, or an apple, or an elementary particle, this mass is indeed not a random number; it is what it is and cannot have a distribution. But when using the Bayesian formalism, we assign such a probability to the parameters. The prior probability corresponds to the state of our knowledge (i.e., belief) about the parameter. This change of interpretation of the symbols regarding probabilities (note that the mathematical axioms of probability do not change under Bayesianism, however !) introduces the notion of posterior probability distribution for parameters.

  Examples of the prior's influence on the posterior distribution are provided in a first part to develop intuition on Bayes' estimation.

  The second part turns to "uninformative"'priors (Laplace, Jeffreys), to the most commonly used Bayesian point estimators (posterior mean, maximum and median) and to credible intervals.

  The chapter ends with a detailed comparison list of frequentist versus Bayesian inference. According to B. Efron, a famous contemporary statistician, Statistical inference at its most successful combines elements of the two philosophies, as for instance in the empirical Bayes methods.