THE SEISMIC SOLAR RADIUS

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HISTORY

- Archimedes (287–212 B.C.) is credited with first approximate measure of apparent diameter of the Sun, between $1/200$ and $1/164$ of a right angle or $27' - 32'\,56''$ (current value is between $31'\,3''$ and $32'\,35''$).

- Jean Picard (1620–1682) was probably the first person to measure solar diameter accurately.

- The currently accepted value of solar diameter was given by Auwers (1891, AN 128, 361) $1919''.26$ (or $R_\odot = 695.99 \pm 0.07$ Mm)

TEMPORAL VARIATIONS IN SOLAR RADIUS

- J. Winlock (1853, AJ 3, 97) argued that the differences in reported values of solar diameter are due to differences in instruments and to the observers.

- Secchi (1872, C. R. Acad Sci. Paris 75, 606) found that solar diameter varies with solar activity by up to $3''$ and is anticorrelated with activity.

- Auwers (1873, MNRAS 34, 22) has argued that reported variation in solar diameter by Secchi are not real. He did not find any correlation with activity in Greenwich observations during 1851–1870.

- More recently Delache (1988, AdSpR 8, 119) argued that even if the variations in the apparent diameter are well established, it is impossible to relate them to variations of the real solar diameter.

- There is no agreement on observed variations in solar radius as reported variations vary from 0 to about 700 km. Even the sign is disputed as some find it correlated to solar activity, while others find anti-correlation.

- Parkinson et al. (1980, Nature 288, 548); Brown & Christensen-Dalsgaard (1998, ApJ 500, L195) found no significant variation in solar radius.

- Laclare et al. (1996, Solar Phys. 166, 211) found variations that are anti-correlated with solar activity.

- Wittman et al. (1993, Solar Phys. 145, 205) and Chapman et al. (2008, ApJ 681, 1698) found variations correlated with solar activity.

- Measurement from MDI instrument (Kuhn et al. 2004, ApJ 613, 1241) found much smaller variations with an upper limit of 7 mas, or about 5 km.

- Even a variation by 1 km in solar radius will cause variation in gravitational energy by

$$\frac{GM_\odot^2}{R_\odot^2}\delta R \approx 5 \times 10^{42}\ \delta R \qquad \text{ergs}$$

where δR is in km. If the variation is over 3 years (10^8 sec) the energy is released or absorbed at the rate of $5 \times 10^{34} \delta R$ ergs per second, which is larger than the solar luminosity.

- Thus radius variation has to be confined to outer layers which have little mass. Even if it extends till the base of the convection zone, the variation will be 2% of the above value.

- If the luminosity variations are limited to 0.1%, then the radius variation has to be confined to outer 10^{-4} of solar mass or outer 4% of solar radius.

- These radius variations are not likely to contain any useful information about variations in the solar interior.

- Definition of Solar surface:
Observers: Point of inflexion in intensity profile ($\tau \approx 0.001$)
Theoretical solar models: Optical depth $\tau \approx 1$

$$4\pi R_\odot^2 \sigma T^4 = L_\odot$$

- The radius used in solar models should be about 500 km less than the observed value (Brown, Christensen-Dalsgaard 1998, ApJ 500, L195)

- Haberreiter et al. (2008, ApJ 675, L53) find a difference of 333 km with respect to MDI continuum measurement. But Kuhn et al. (2004) find a radius of 695.74 ± 0.11 Mm from MDI.

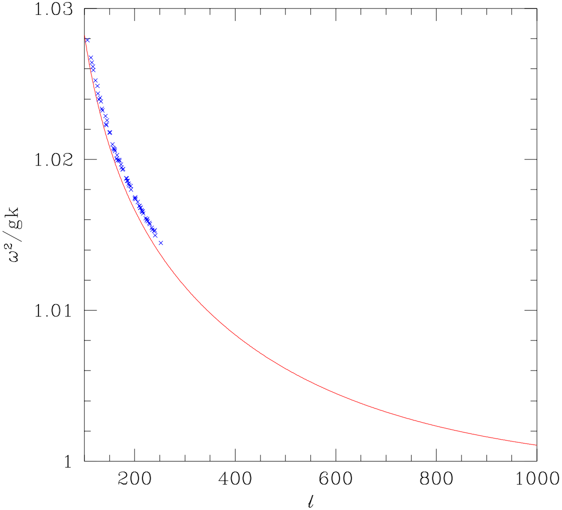
Seismic estimate of Solar Radius

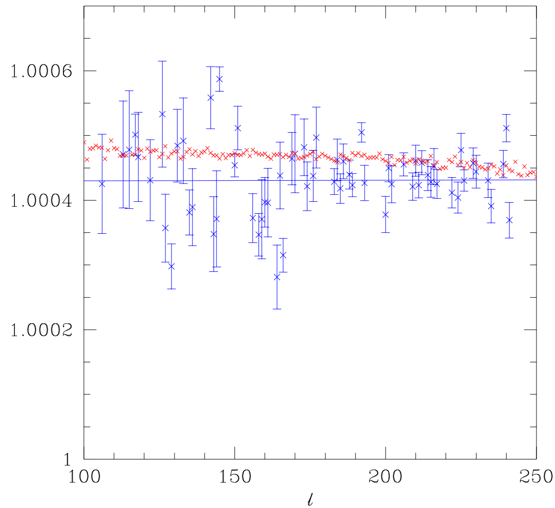
- **f-modes:** Surface gravity modes

- Frequencies are sensitive to surface gravity

$$\omega^2 \approx gk = \frac{GM}{r^2}\frac{\sqrt{\ell(\ell+1)}}{r}$$

  This can be used to calibrate the model radius (Schou et al. 2007, ApJ 489, L197)

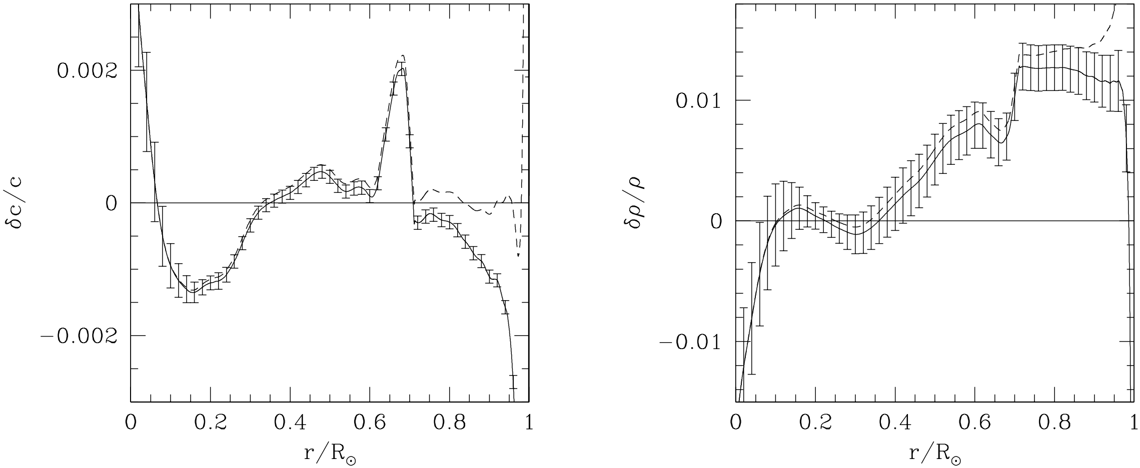




- $\dfrac{\omega^2_{\text{obs}}}{\omega^2_{\text{model}}} \approx 1.0009$

  If this discrepancy is due to error in radius then $\delta R/R = -0.0003$, or $\delta R = -210$ km.

Antia (1998, A&A 330, 336)



Relative difference in sound speed and density between the Sun and the standard solar model of Christensen-Dalsgaard et al. (1996, Science 272, 1286) is shown by *solid* lines with error bars. The *dashed* lines show the results obtained when the radial distance in the solar model is scaled by a factor of 1.00018 before taking the difference

- There are systematic errors of up to 100 km in the seismic estimate arising from uncertainties in stratification of outer layers.

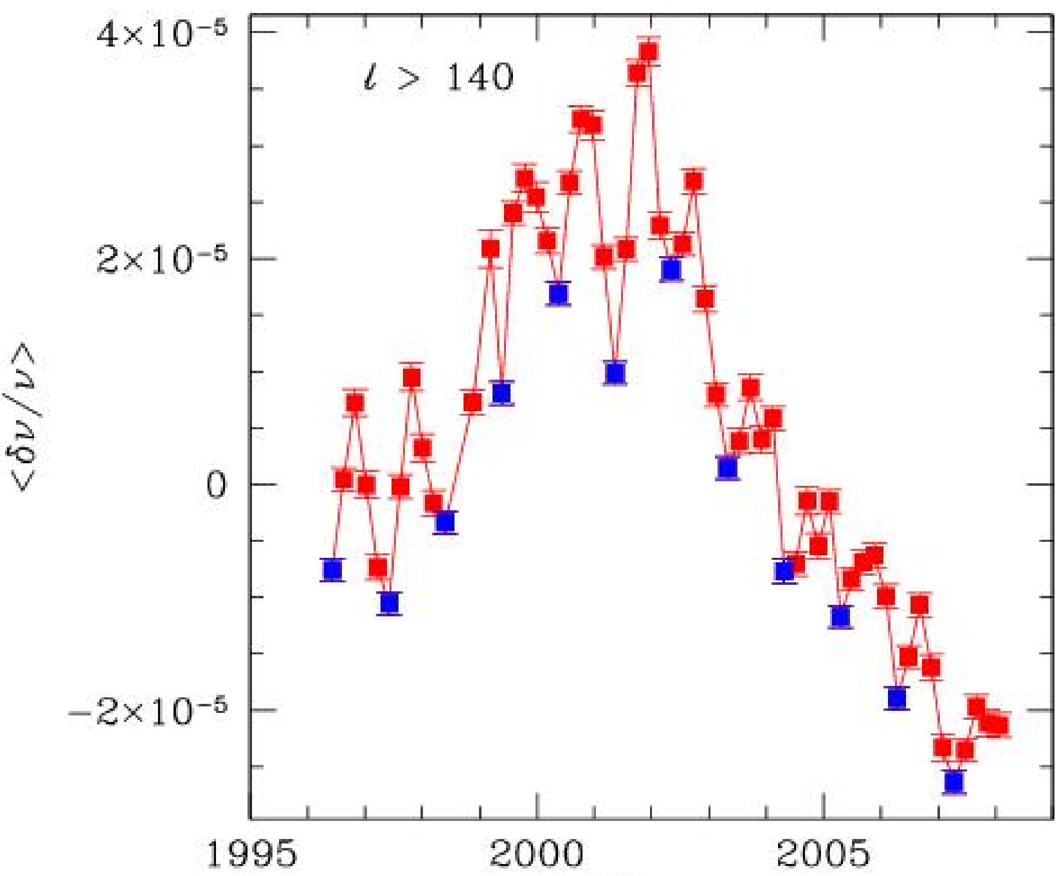
- If systematic errors are independent of time the time variation of radius can be detected at the level of 1 km.

- Even a 1% change in systematic error can give radius variations of order of 1 km. Hence it is difficult to interpret such changes. E.g., change in convective flux with solar cycle can change the calibrated solar radius.

Temporal variations in the seismic solar radius

- From f-mode frequency variation also conflicting results are obtained about radius variations, with variation between 0 to 5 km.

- The reason for this discrepancy is that the observed variation in f-mode frequencies is rather complex, with a significant fraction coming from systematic errors in frequency measurements.



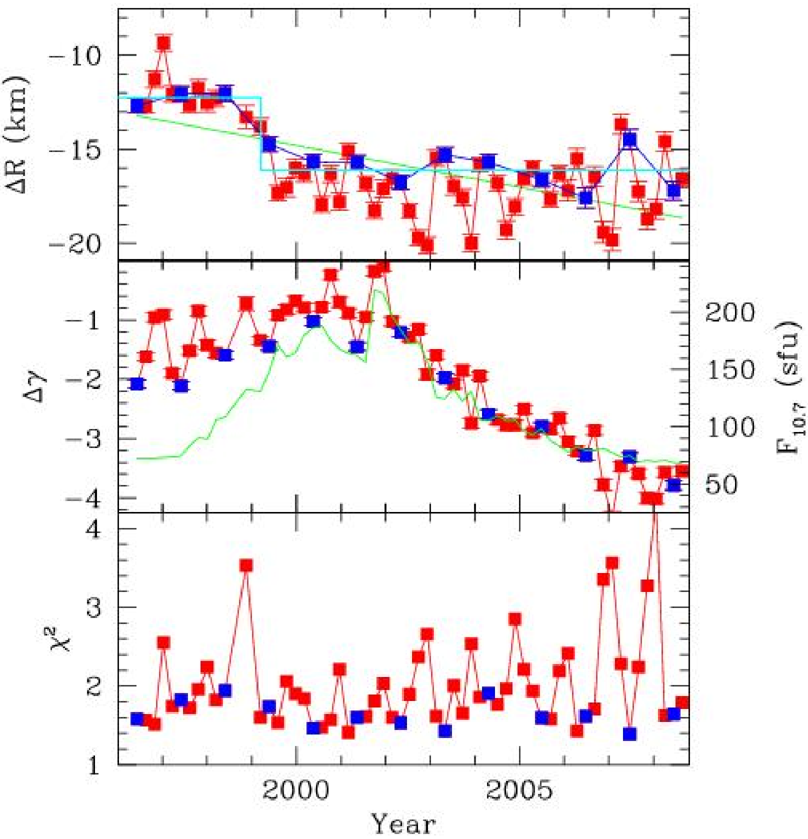
$\ell > 140$
4×10^{-5}
2×10^{-5}
0
-2×10^{-5}
$\langle\delta\nu/\nu\rangle$
1995
2000
2005

- It is difficult to remove the 1 year periodicity by averaging over MDI data as the MDI data sets cover 72 days, giving 360 days for 5 sets, which would accumulate a shift of 63 days over 12 years.

- Temporal variations in f-mode frequencies can be due to radius variation and surface effects (Dziembowski et al. 2001; ApJ 553, 897)

$$\Delta\nu_\ell = -\frac{3}{2}\frac{\Delta R}{R}\nu_\ell + \frac{\Delta\gamma}{I_\ell}$$

- From this analysis it was concluded that the solar radius is decreasing at the rate of 1.5 ± 0.3 km per year.



ΔR (km)
−10
−15
−20
Δγ
−1
−2
−3
−4
200
150
100
50
$F_{10.7}$ (sfu)
χ^2
4
3
2
1
2000
2005
Year

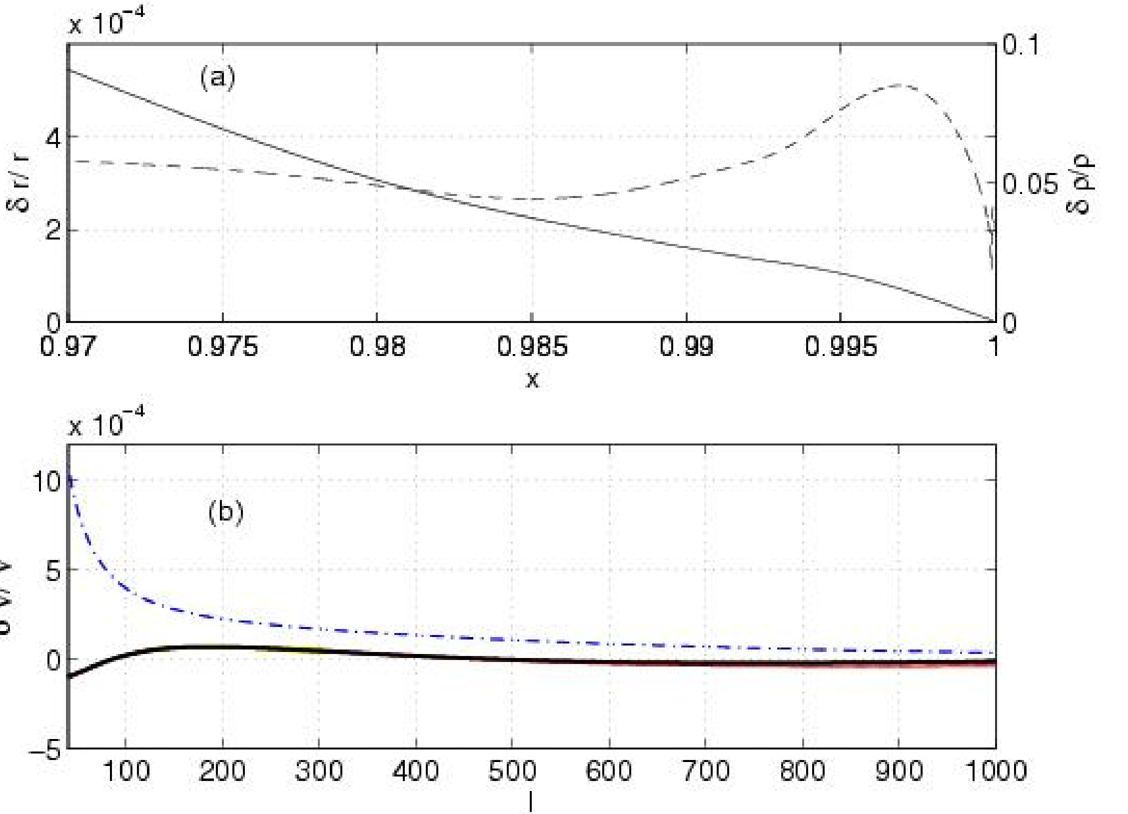
- Dziembowski & Goode (2004, ApJ 600, 464) derived the following relation connecting f-mode frequency change and radius variation

$$\frac{\delta\omega_\ell}{\omega_\ell} = -\frac{3\ell}{2\omega_\ell^2 I_\ell} \int_0^R \frac{g}{r}\frac{\delta r}{r}\, dI_\ell$$

  This assumes that the radius variation is non-homologous.

- This relation has been used by Lefebvre et al. (2005, 2007, ApJ 633, L149; 658, L138) to infer the temporal variation in solar radius at various depths.

- However, this relation has not been tested and a test by Chatterjee & Antia (2008, ApJ 688, L123) using a few pairs of solar model shows that the relation is not valid.

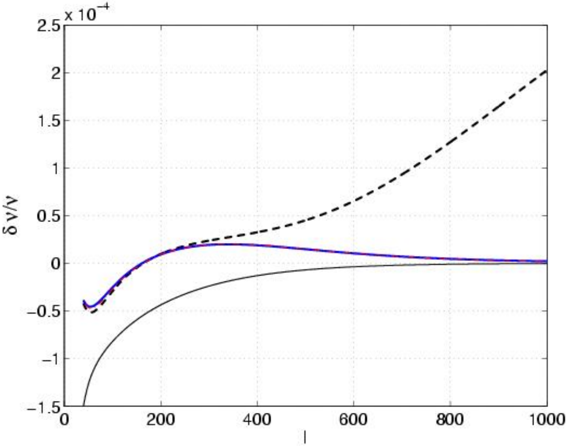


x 10^-4
(a)
δ r / r
δ ρ/ρ
0.1
0.05
0
4
2
0
0.97
0.975
0.98
0.985
0.99
0.995
1
x
x 10^-4
(b)
δ ν/ ν
10
5
0
−5
100
200
300
400
500
600
700
800
900
1000
l

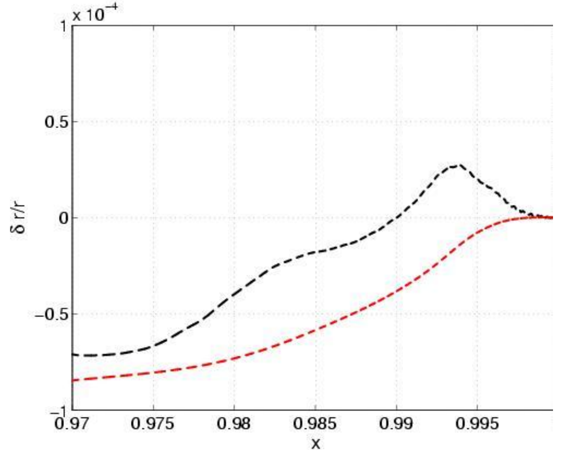
- Lefebvre et al. (2008, ApJ in press) also found similar results, but they also found that if δr is calculated at constant H_p instead of constant mass the relation appears to be valid.

- This can only be a coincidence as the relation has been derived using the definition at constant mass and in fact tests do show significant differences in many model pairs at high degree.

- There will also be a problem in defining solar radius at constant H_p near the surface, as its minimum value in solar model may be different in different models.



x 10^−4
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
δ ν/ν
0
200
400
600
800
1000
l



x 10^−4
1
0.5
0
−0.5
−1
δ r/r
0.97
0.975
0.98
0.985
0.99
0.995
x

- In general a perturbation in density can be interpreted as radius variation through

$$\frac{\delta\rho}{\rho} = -\frac{1}{r^2}\frac{d}{dr}\left(r^2\delta r\right) + \frac{\delta r}{H_\rho}$$

  However, the sound speed perturbation possibly cannot be expressed entirely in terms of radius variation.

- Thus any structure variation in solar model can be expressed in terms of δr and δc^2, but it is not clear how this will help in interpreting structure changes.

- When radius variations are not homologous there is also the question of its definition. Dziembowski & Goode (2004) have used variation in r at constant mass, but it is not clear if that definition is meaningful in interpreting observed variation as these will correspond to some other definition.

- There may be no simple relation connecting the two values of δr. A detailed atmospheric model will be needed to translate from one definition to another.

CONCLUSIONS

- More studies are needed to resolve the discrepancy between the observed solar radius and the seismic radius as estimated by calibrating solar models to match the f-mode frequencies.

- Any possible variation in the solar radius has to be confined to outer layers.

- It is very difficult to interpret observed variation in radius at the solar surface in terms of radius variation in solar models.

- F-mode frequencies can be affected by a number of factors other than solar radius, e.g., convection, magnetic field etc. Hence, it is not possible to use these frequencies to infer radius variations in the solar interior. Some of the relations used in these studies are not even valid.

- Apart from these, there are many systematic errors in observed f-mode frequencies which make it difficult to identify the true variations in these frequencies.