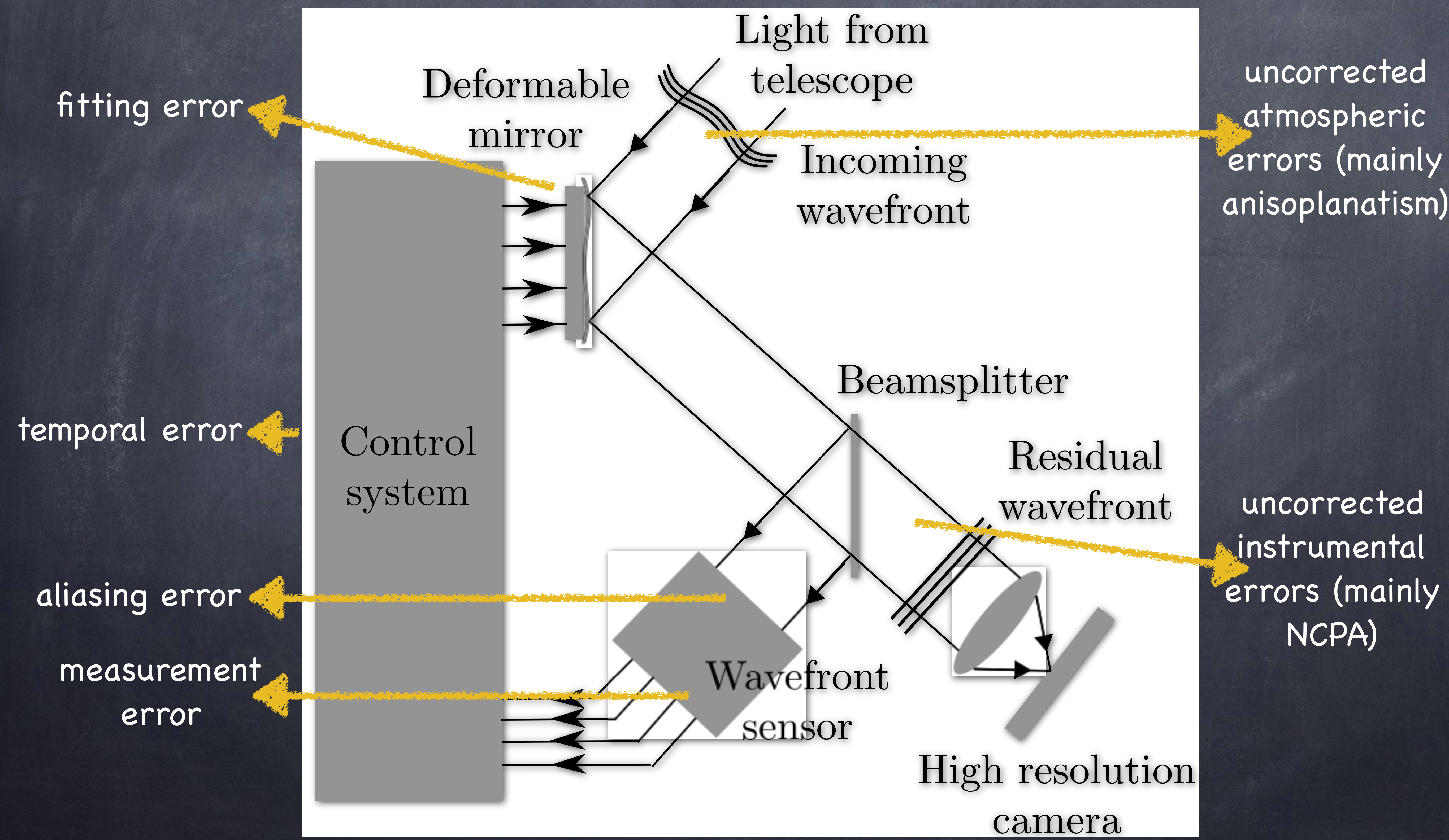
Post-AO error budget - 1



Post-AO error budget - 2

Global error (variance on the corrected phase/wavefront)

$$\sigma^2_{\mathrm{post-AO}} = \sigma^2_{\mathrm{atm.}} + \sigma^2_{\mathrm{AO\ syst.}} + \sigma^2_{\mathrm{others}}$$

- **Error term due to turbulent atmosphere alone**
- **Residual error of the AO system itself**
- **Other error terms…**

Post-AO error budget - 3

Other errors

$$\boxed{\sigma^2_{\text{others}} = \sigma^2_{\text{NCPA}} + \sigma^2_{\text{calib.}} + \ldots}$$

- **Error term due to Non-Common Path Aberrations**
- **Error term due to the calibration of the AO system**
- **etc.**

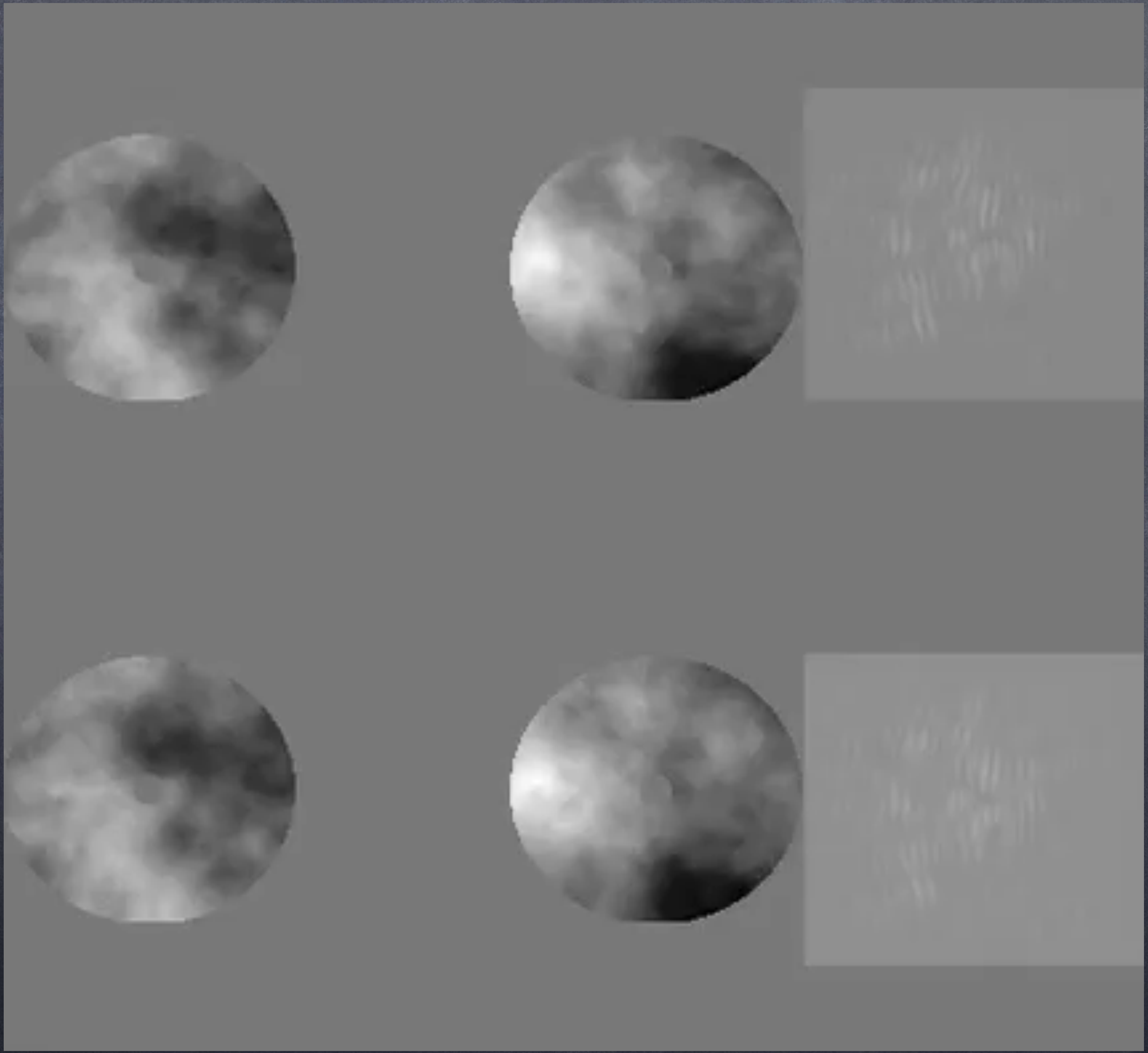
Post-AO error budget - 4

Error due to atmospheric turbulence alone

$$\boxed{\sigma^2_{\mathrm{atm.}} = \sigma^2_{\mathrm{aniso}} + \sigma^2_{\mathrm{scint..}} + \sigma^2_{\mathrm{diff.}} + \sigma^2_{\mathrm{chrom.}}}$$

- <u>**Error term due to anisoplanatism**</u>
- **Error term due to scintillation**
- **Error term due to diffractive effects**
- **Error term due to differential refraction**

Post-AO error budget - 5



Post-AO error budget - 6

Residual error of the AO system itself

$$\sigma^2_{\text{AO syst.}} = \sigma^2_{\text{fitt.}} + \sigma^2_{\text{meas.}} + \sigma^2_{\text{alias.}} + \sigma^2_{\text{temp.}} + \dots$$

$$\dots + \sigma^2_{\text{LGS}} + \sigma^2_{\text{MCAO}}$$

- Fitting error (due to spatial under-sampling of the DM)
- Measurement error (due to photon noise, RON, etc. - WFS)
- Aliasing error (due to spatial under-sampling of the WFS)
- Temporal error (due to finite temporal bandwidth of the whole system)

- Specific errors of the LGS
- Specific errors of the MCAO (et similia)

Post-AO error budget - 7

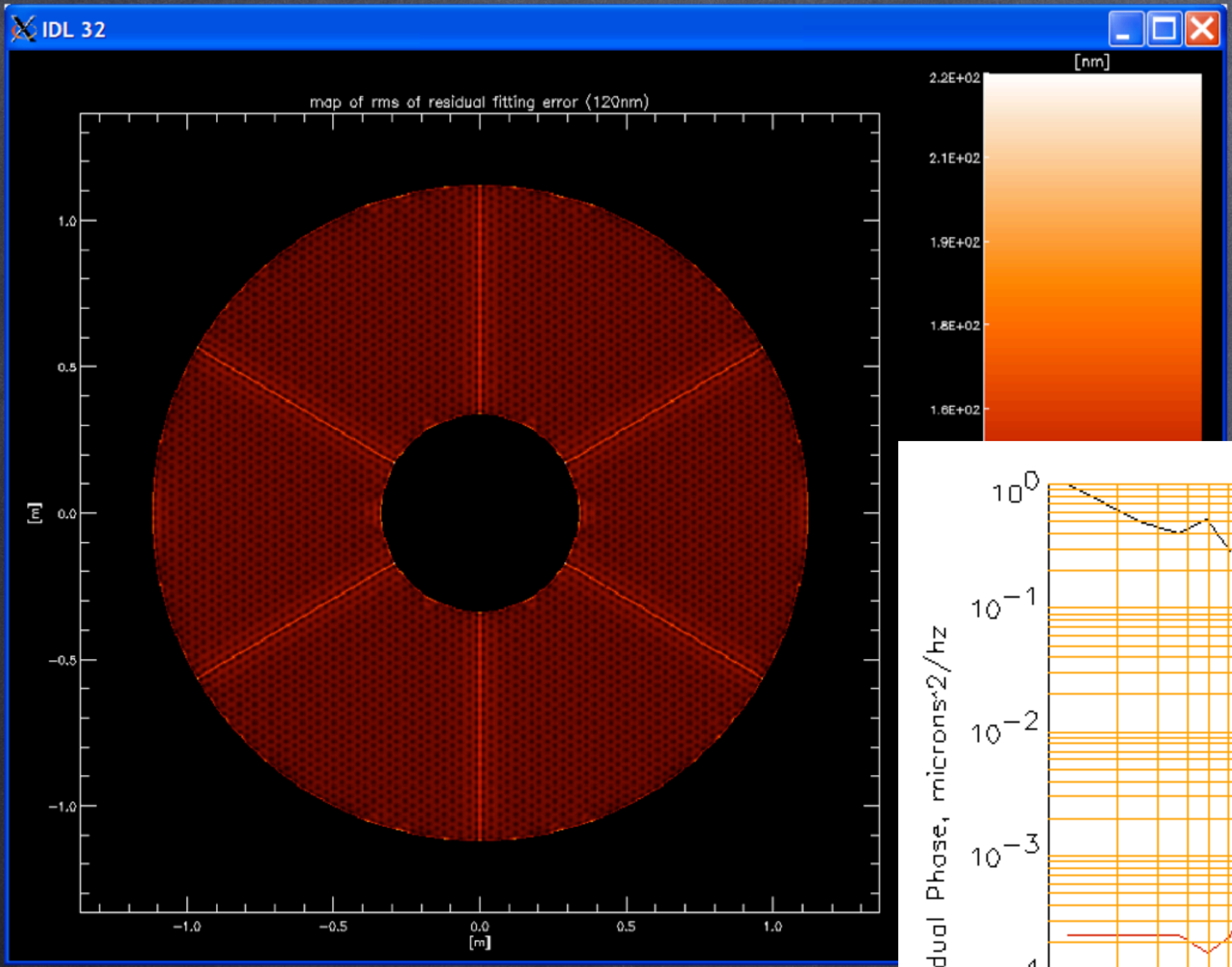
Fitting error

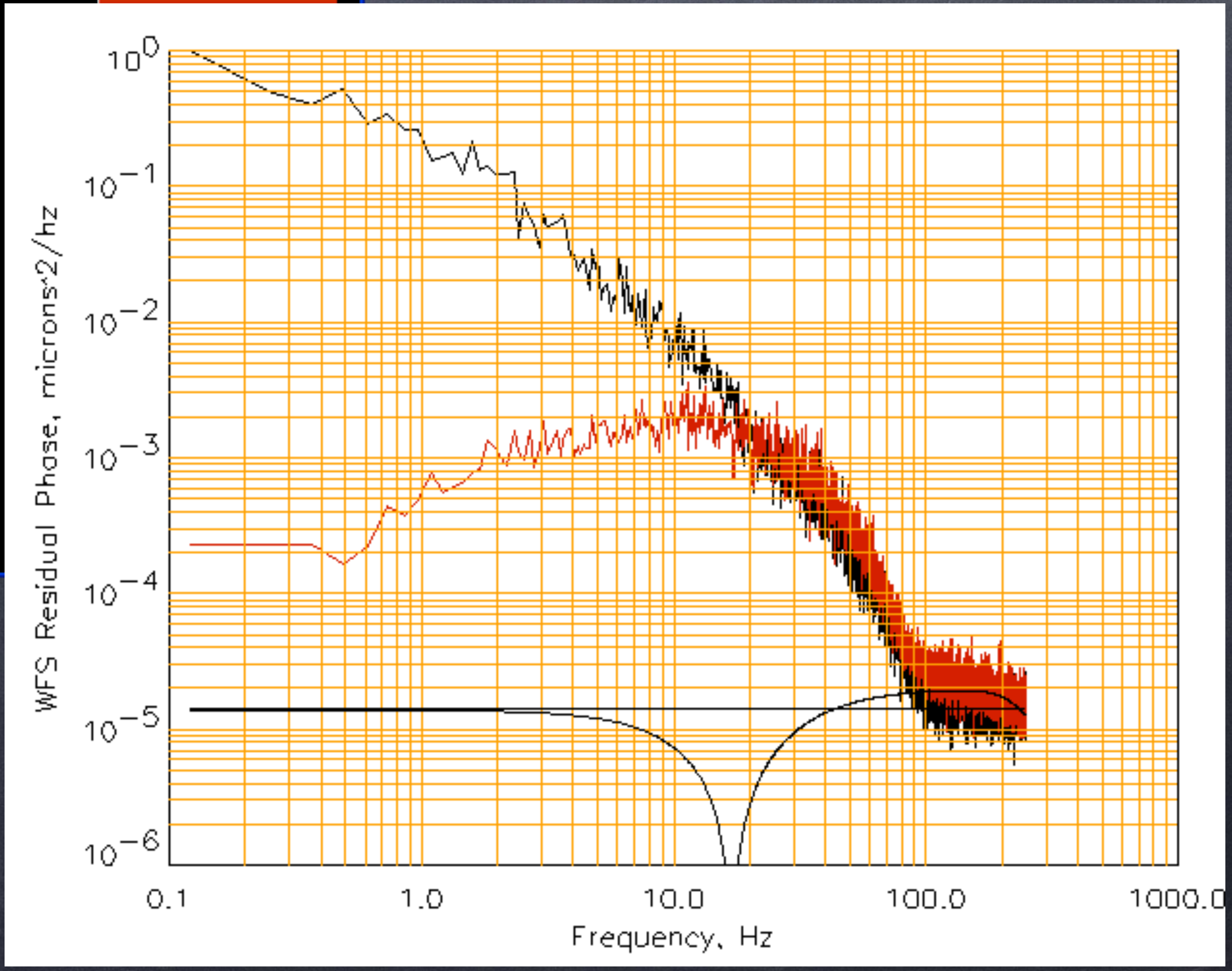
$$\sigma^2_{\text{AO syst.}} = \sigma^2_{\text{fitt.}} + \sigma^2_{\text{meas.}} + \sigma^2_{\text{alias.}} + \sigma^2_{\text{temp.}} + \ldots$$

$$\sigma^2_{\text{fitt.}} \propto \left(\frac{d}{r_0}\right)^{\frac{5}{3}}$$

Reduce the fitting error <=> increase the number of actuators of the DM…

Post-AO error budget - 8





(Noll: residual error - 1)

If, instead of having actuators, one could have a mirror capable of forming perfect Zernike polynomials, one would have (admitting that atmosphere exactly follows a Kolmogoroff model):

$$\boxed{\Delta_J \simeq 0.2944\ J^{-\sqrt{3}/2} \left(\frac{D}{r_0}\right)^{5/3}, J \geq 20}$$

Hence, in meters:

$$\boxed{\sigma_J[m] \simeq \frac{\lambda}{2\pi}\sqrt{\Delta_J} \simeq 0.352\ J^{-\sqrt{3}/4}\ D^{5/6}\ \left(\int_0^\infty C_n^2(z)\ dz\right)^{1/2}}$$

With, thanks to Maréchal's approximation:

$$\boxed{S \simeq \exp\{-\Delta\}}$$

(Noll: residual error - 2)

TABLE IV. Zernike-Kolmogoroff residual errors (Δ_J). (D is the aperture diameter.)

$\Delta_1 = 1.0299\ (D/r_0)^{5/3}$	$\Delta_{12} = 0.0352\ (D/r_0)^{5/3}$
$\Delta_2 = 0.582\ (D/r_0)^{5/3}$	$\Delta_{13} = 0.0328\ (D/r_0)^{5/3}$
$\Delta_3 = 0.134\ (D/r_0)^{5/3}$	$\Delta_{14} = 0.0304\ (D/r_0)^{5/3}$
$\Delta_4 = 0.111\ (D/r_0)^{5/3}$	$\Delta_{15} = 0.0279\ (D/r_0)^{5/3}$
$\Delta_5 = 0.0880\ (D/r_0)^{5/3}$	$\Delta_{16} = 0.0267\ (D/r_0)^{5/3}$
$\Delta_6 = 0.0648\ (D/r_0)^{5/3}$	$\Delta_{17} = 0.0255\ (D/r_0)^{5/3}$
$\Delta_7 = 0.0587\ (D/r_0)^{5/3}$	$\Delta_{18} = 0.0243\ (D/r_0)^{5/3}$
$\Delta_8 = 0.0525\ (D/r_0)^{5/3}$	$\Delta_{19} = 0.0232\ (D/r_0)^{5/3}$
$\Delta_9 = 0.0463\ (D/r_0)^{5/3}$	$\Delta_{20} = 0.0220\ (D/r_0)^{5/3}$
$\Delta_{10} = 0.0401\ (D/r_0)^{5/3}$	$\Delta_{21} = 0.0208\ (D/r_0)^{5/3}$
$\Delta_{11} = 0.0377\ (D/r_0)^{5/3}$	

$\Delta_J \sim 0.2944\, J^{-\sqrt{3}/2}\ (D/r_0)^{5/3}$ (For large J)

(Noll: residual error - 3)

Exercice: Which mirror configuration for a (maximum) goal Strehl ratio of 30% in band J ? [r0=10cm, D=8m]

(Noll: residual error - 4)

- Fried parameter in band J:

$$r0[J] = 0.1\ (1.25/0.5)^{6/5} = 0.3$$

- What we want is hence:

$$0.3 = \exp\{-0.2944\ J^{-\mathrm{sqrt}(3)/2}\ (D/r0)^{5/3}\}$$

(Thanks to Maréchal and Noll...)

Then: $J = 109$ (minimum)

- But: $J = (N+1)(N+2)/2-1 \Rightarrow 13<N<14$

Hence: $N=14$ (which corresponds to $J=119$) in order to have the minimum required...

(Noll: residual error - 5)

Exercice > report : Compute the Noll error and then the corresponding maximum Strehl ratio in J for a 10x10 AO system [D=1m, r0@500nm=10cm]

Post-AO error budget - 9

Aliasing error

$$\sigma^2_{\mathrm{AO\ syst.}} = \sigma^2_{\mathrm{fitt.}} + \sigma^2_{\mathrm{meas.}} + \sigma^2_{\mathrm{alias.}} + \sigma^2_{\mathrm{temp.}} + \ldots$$

$$\sigma^2_{\mathrm{alias.}} \propto \left(\frac{d}{r_0}\right)^{\frac{5}{3}}$$

Reduce the aliasing error <=> increase the number of sensing elements (Shack-Hartmann sub-apertures, pixels of the pyramid) within the WFS

Post-AO error budget - 10

Temporal error

$$\sigma^2_{\text{AO syst.}} = \sigma^2_{\text{fitt.}} + \sigma^2_{\text{meas.}} + \sigma^2_{\text{alias.}} + \sigma^2_{\text{temp.}} + \ldots$$

$$\sigma^2_{\text{temp.}} \propto \left(\frac{\Delta t_{\text{AO}}}{\tau_0} \right)^{\frac{5}{3}}$$

Reduce the temporal error <=> make a faster system (exposure time of the WFS, computing time for the wavefront reconstruction, actuating time for the DM)

Post-AO error budget - 11

Measurement error

$$\sigma^2_{\mathrm{AO\ \ syst.}} = \sigma^2_{\mathrm{fitt.}} + \sigma^2_{\mathrm{meas.}} + \sigma^2_{\mathrm{alias.}} + \sigma^2_{\mathrm{temp.}} + \ldots$$

$$\sigma^2_{\mathrm{mes.}} = \sigma^2_{\mathrm{phot.}} + \sigma^2_{\mathrm{RON}} + \ldots$$

The measurement error has many origins:

- **photon noise**
- **read-out noise (RON)**
- **dark-current noise**
- **sky background and possibly instrumental background**
- **in case of EMCCD: (almost) no RON but additional noises (exotic dark, « excess noise factor » => Gamma-distributed noise)**

Post-AO error budget - 12

Photon noise error term

$$\sigma^2_{\text{AO syst.}} = \sigma^2_{\text{fitt.}} + \sigma^2_{\text{meas.}} + \sigma^2_{\text{alias.}} + \sigma^2_{\text{temp.}} + \ldots$$

$$\sigma^2_{\text{mes.}} = \sigma^2_{\text{phot.}} + \sigma^2_{\text{RON}} + \ldots$$

$$\sigma^2_{\text{phot.}} \propto \frac{1}{N_{\text{phot.}}}$$

(with N_{phot}=number of photons/exposure time/subaperture)

Reduce the photon noise error term <=>
1- reduce the number of WFS elements => increase the aliasing error !!
2- increase the exposure time => increase the temporal error !!

Post-AO error budget - 13

Read-out noise error term

$$\sigma^2_{\mathrm{AO\ syst.}} = \sigma^2_{\mathrm{fitt.}} + \sigma^2_{\mathrm{meas.}} + \sigma^2_{\mathrm{alias.}} + \sigma^2_{\mathrm{temp.}} + \ldots$$

$$\sigma^2_{\mathrm{mes.}} = \sigma^2_{\mathrm{phot.}} + \sigma^2_{\mathrm{RON}} + \ldots$$

$$\sigma^2_{\mathrm{lect.}} \propto \frac{\sigma_e}{N^2_{\mathrm{phot.}}}$$

Reduce the RON error term <=>
1- reduce the number of WFS elements => increase the aliasing error !!
2- increase the exposure time => increase the temporal error !!
3- but also: reduce the impact of RON => use of EMCCDs…

Post-AO error budget - 14

Generic case: observe as much sources as possible
Problem: most sources are (obviously!) too faint

1- find and use brighter NGS nearby…
=> anisoplanatic error !
=> use more than one brighter NGS nearby…
=> multi-reference AO system (GLAO, MCAO, MOAO)
=> yes, but: specific errors !
=> limited quality of correction

2- create a brighter source (LGS)
=> 100% sky coverage
=> yes, but here again: specific errors !
=> limited quality of correction