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r0 en bande H sachant r0 à 500nm ?...

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^{\infty} C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

$$r_0^{\mathrm{H}=1.65\,\mu\mathrm{m}} = r_0^{500\,\mathrm{nm}} \left(\frac{1.65}{0.5}\right)^{\frac{6}{5}} \simeq 0.42$$

Nombre de speckles pour r0=10cm et D=1m ?...

$$N_S^{500\,\mathrm{nm}} \simeq 0.34\ \left(\frac{1.0}{0.1}\right)^2 \simeq 34$$

$$N_S^{\mathrm{H}} \simeq 0.34\ \left(\frac{1.0}{0.42}\right)^2 \simeq 2$$

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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Modèle de Kolmogorov/von Kármán

- Kolmogorov : échelle externe $\mathcal{L}_0$ infinie.
- On peut affiner en considérant aussi l'échelle interne ℓ_0.
- $\exists$ d'autres modèles avec $\mathcal{L}_0$ finie et ℓ_0 non-nulle.

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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Numériquement, et en considérant des fronts d'onde de côté « dim » pixels correspondant à « L » mètres, ceci s'écrit :

```
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

Ce qui, avec la bonne échelle en fréquences, peut se tracer :

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

=> tracer la DSP pour différents [r_0, L_0]…

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-> Génération de fronts d'onde perturbés par la turbulence atmosphérique...

The well-known FFT method allows us to generate phase screens $\varphi(\vec{r})$, where $\vec{r}$ is the two-dimensional position within the phase screen, assuming usually either a Kolmogorov or a von Karman spectrum $\Phi_\varphi(\vec{\nu})$, where $\vec{\nu}$ is the two-dimensional spatial frequency, from which is computed the modulus of $\tilde{\varphi}(\vec{\nu})$, the Fourier transform of $\varphi(\vec{r})$. Assuming the near-field approximation and small phase perturbation [3], the von Karman/Kolmogorov spectrum is given by

$$\Phi_\varphi(\vec{\nu}) = 0.0229 r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}, \tag{1}$$

where r_0 is the Fried parameter and $\mathcal{L}_0$ is the wavefront outer scale (infinite for the Kolmogorov model). Within the framework of the classical FFT-based technique, a turbulent phase screen $\varphi_L(\vec{r})$ of physical length L is obtained by taking the inverse FFT of $\tilde{\varphi}_L(\vec{\nu})$, the modulus of which is obtained from Eq. (1) by applying the definition of the power spectrum, which is

$$\Phi_\varphi(\vec{\nu}) = \lim_{L\to\infty}\left(\frac{\langle|\tilde{\varphi}_L(\nu)|^2\rangle}{L^2}\right)$$

$$\Rightarrow |\tilde{\varphi}_L(\nu)| \simeq L r_0^{-\frac{5}{6}} \sqrt{0.0228}\left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{12}}, \tag{2}$$

and which phase is random and uniformly distributed.

(la même manipulation que précédemment est ici appliquée afin d'obtenir la formulation numérique ci-dessous.)

The obtained phase screen is thus numerically written

$$\varphi_L(i,j) = \sqrt{2}\sqrt{0.0228}\left(\frac{L}{r_0}\right)^{\frac{5}{6}}\left\{\mathrm{FFT}^{-1}\left[\left(k^2 + l^2 + \left(\frac{L}{\mathcal{L}_0}\right)^2\right)^{-\frac{11}{12}} \exp\{\imath\theta(k,l)\}\right]\right\}, \tag{3}$$

where i and j are the indices in the direct space, k and l are the indices in the FFT space, $\{\}$ stands for either *real part of* or *imaginary part of*, $\imath$ is the imaginary unit, and θ is the random uniformly distributed phase (between $-\pi$ and π). The factor $\sqrt{2}$ comes from the fact that here we use both the real and imaginary parts of the original complex generated FFT phase screens, which are independent one from the other [4]. This kind of phase screen suffers, however, from the lack of spatial frequencies lower than the inverse of the necessarily finite length L of the simulated array.

```
function wfgeneration, dim, length, L0, r0, lambda, SEED=seed
;
; wave-front (wf) generation following Kolmogorov or von Karman model
;
; dim    = wf linear dimension [px],
; length = wf physical length [m],
; L0     = wf outer-scale [m],
; seed   = random generation seed (OPTIONAL),
; r0     = Fried parameter at wavelength 'lambda' [m],
; lambda = wavelength at which r0 is defined.
;
; Marcel Carbillet [marcel.carbillet@unice.fr],
; UMR 7293 Lagrange (UNS/CNRS/OCA), February 2013.
;
; Last modification: Feb. 2014
;
phase = (randomu(seed,dim,dim)-.5) * 2*!PI    ; rnd uniformly distributed phase
                                              ; (between -PI and +PI)
rr = dist(dim)
if L0 eq !VALUES.F_INFINITY then rr[0,0] = 1.; avoid 1/infinity afterwards
                                              ; for Kolmogorov model
modul = (rr^2+(length/L0)^2)^(-11/12.)        ; vonKarman/Kolmogorov model
if L0 eq !VALUES.F_INFINITY then modul[0,0] = 0.
                                              ; put again modulus[0,0]=0
                                              ; for Kolmogorov model
screen = fft(modul*exp(complex(0,1)*phase), /INVERSE)
                                              ; compute wf
screen *= sqrt(2)*sqrt(.0228)*(length/r0)^(5/6.)*lambda/(2*!PI)
                                              ; proper nor
                                              ; (consideri
screen -= mean(screen)

return, screen                                ; deliver 2
                                              ; float(scre
end
```

<u>« wf generation »</u>
générer un cube de fronts d'onde statistiquement indépendants (une centaine)… => calculer l'écart-type moyen pour différents [r_0, L_0]